NEW YORK STATE TEACHER CERTIFICATION EXAMINATIONS™

FIELD 004: MATHEMATICS

TEST DESIGN AND FRAMEWORK

May 2018

Authorized for Distribution by the New York State Education Department

This test design and framework document is designed to provide information about the content and format of a test for the New York State Teacher Certification Examinations™ (NYSTCE®) program. Education faculty and administrators at teacher preparation institutions may also find the information in this framework useful as they discuss the test with candidates. All test components may differ from those presented here. Furthermore, review of this framework, in whole or in part, does not guarantee an increased likelihood of success on any of the New York State Teacher Certification Examinations. The NYSTCE program is subject to change at the sole discretion of the New York State Education Department, and any changes will fully supersede the information presented in this document. As a reminder, candidates are responsible for contacting their certification officer(s) regarding any changes to the New York State Teacher Certification Examinations.

NEW YORK STATE TEACHER CERTIFICATION EXAMINATIONS™

FIELD 004: MATHEMATICS

TEST DESIGN

This test consists of selected-response items measuring content knowledge and one extended constructed-response item measuring pedagogical content knowledge. The constructed-response item is scenario-based and requires candidates to describe an instructional strategy to help students achieve a specific learning goal or an instructional intervention to address a specific learning difficulty, and to provide a rationale for employing that instructional strategy or intervention.

The selected-response items count for 80% of the total test score and the constructed-response item counts for 20% of the total test score, as indicated in the table that follows. Each selected-response item counts the same toward the total test score. The percentage of the total test score derived from the constructed-response item is also indicated in the table that follows.

The total testing time is 210 minutes. Candidates are free to set their own pace during the test administration. The following estimates were used to determine the total test time:

- The constructed-response item is designed with the expectation of a response up to 60 minutes.
- The selected-response items are designed with the expectation of response time up to 150 minutes.

Further information regarding the content of each competency can be found in the test framework.

FIELD 004: MATHEMATICS
TEST DESIGN

Competency	Selected-Response		Constructed-Response	
	Approximate Number of Items	Approximate Percentage of Test Score	Number of Items	Approximate Percentage of Test Score
0001 Number and Quantity	9	8%	--	--
0002 Algebra	23	20%	--	--
0003 Functions	19	17%	--	--
0004 Calculus	11	10%	--	--
0005 Geometry and Measurement	17	15%	--	--
0006 Statistics and Probability	11	10%	--	--
0007 Pedagogical Content Knowledge	--	--	1	20%
Total	**90**	**80%**	**1**	**20%**

Note:

- There is no calculus component in the New York State Learning Standards for Mathematics. In the existing NYSTCE® Mathematics test, calculus represents approximately 10% of the framework, assuming natural weighting based on the number of competencies.

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FIELD 004: MATHEMATICS
TEST FRAMEWORK

Number and Quantity
Algebra
Functions
Calculus
Geometry and Measurement
Statistics and Probability
Pedagogical Content Knowledge

The New York State Mathematics educator has the knowledge and skills necessary to teach effectively in New York State classrooms. A mathematics teacher has a deep understanding of number and quantity, algebra, functions, calculus, geometry and measurement, statistics and probability, and modeling and demonstrates strong mathematical practices: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, modeling with mathematics, making connections across mathematical domains, using appropriate tools strategically, attending to precision, looking for and making use of structure, and looking for and expressing regularity in repeated reasoning.

COMPETENCY 0001—NUMBER AND QUANTITY

Performance Expectations

The New York State Mathematics teacher understands and extends concepts of number and quantity, from the properties of arithmetic operations involving real numbers through the properties of operations involving vector and matrix representations and complex numbers.

Performance Indicators

a. applies and extends understanding of arithmetic to the rational numbers
b. applies properties of rational numbers to solve real-world and mathematical problems involving the four operations with rational numbers
c. applies and extends understanding of integer exponents to include rational exponents and rewrites expressions involving radicals and rational numbers
d. reasons quantitatively and uses appropriate units to solve problems
e. demonstrates understanding of the properties of real numbers and applies real numbers to model and solve multistep problems

f. performs arithmetic operations with complex numbers

g. represents complex numbers and their operations in the complex plane, using both rectangular and polar forms

h. uses complex numbers to factor and solve quadratic equations and applies the fundamental theorem of algebra

i. models and solves problems with vector quantities, including problems involving velocity and other quantities represented by vectors

j. performs arithmetic operations (e.g., addition, subtraction, scalar multiplication) on vectors and represents vectors, their magnitudes, and vector operations symbolically and graphically

k. demonstrates understanding of the properties of matrices, performs operations on matrices, and uses matrices in applications

l. demonstrates knowledge of abstract algebra (e.g., groups, rings, fields, vector spaces)

COMPETENCY 0002—ALGEBRA

Performance Expectations

The New York State Mathematics teacher understands the use of numbers, symbols, operations, and conventions of notation that allow the creation, interpretation, and manipulation of algebraic expressions and equations, and uses them to model and solve mathematical and real-world problems.

Performance Indicators

a. uses properties of operations to generate equivalent expressions and solves real-life and mathematical problems using numerical and algebraic expressions and equations

b. analyzes rates and proportional relationships and uses them to solve real-world and mathematical problems

c. analyzes connections between proportional relationships, lines, and linear equations

d. interprets the structure of expressions and rewrites expressions in equivalent forms (e.g., factoring, completing the square in a quadratic expression, transforming exponential expressions and equations, finding the sum of a finite geometric series)

e. performs arithmetic operations on polynomials, simplifies polynomial expressions using identities, and expands binomials

f. demonstrates understanding of the relationship between zeros and factors of polynomials and extends polynomial identities to the complex numbers

g. rewrites and manipulates rational expressions

h. creates equations and inequalities in one, two, or more variables to describe numbers or relationships (e.g., linear, quadratic, exponential), including situations involving constraints, and interprets the viability of options in modeling contexts

i. understands solving equations and inequalities as a process of reasoning and explains the reasoning, including situations when extraneous solutions may arise

j. solves linear equations and inequalities and quadratic equations in one variable

k. solves systems of linear and quadratic equations using a variety of methods (e.g., algebraic, graphic, matrix)

l. represents and solves linear and nonlinear equations and inequalities graphically

COMPETENCY 0003—FUNCTIONS

Performance Expectations

The New York State Mathematics teacher understands that functions are descriptions, often in the form of algebraic expressions, of situations in which one quantity depends on another, and that functions have many applications modeling nature and human society.

Performance Indicators

a. demonstrates understanding of the concept of a function and the use of function notation, including sequences and recursive functions

b. interprets functions that arise in applications in terms of context and interprets key features (e.g., domain, intercepts, rate of change, end behavior, periodicity) of functional relationships presented in written descriptions, symbolic expressions, tables, or graphs

c. uses different representations to analyze functions (e.g., linear, quadratic, radical, rational, piecewise, absolute value, exponential, logarithmic)

d. builds functions that model relationships between two quantities using a variety of methods (e.g., explicit expressions, recursive processes, arithmetic combination of functions, composition of functions)

e. analyzes arithmetic and geometric sequences both recursively and with an explicit formula, translates between the two forms, and uses them to model situations

f. builds new functions from existing functions and analyzes their graphs (e.g., analyzes the effect of replacing $f(x)$ with $f(x) + k$, $kf(x)$, $f(kx)$ $f(x + k)$), finds and analyzes inverse functions, and identifies even and odd functions

g. compares and contrasts linear, quadratic, and exponential functions, and uses them to model and solve problems

h. solves problems involving logarithmic and exponential functions

i. analyzes trigonometric functions using the unit circle

j. models periodic phenomena with trigonometric functions

k. proves and applies trigonometric identities

l. solves trigonometric equations

COMPETENCY 0004—CALCULUS

Performance Expectations

The New York State Mathematics teacher understands the fundamental concepts of calculus and how techniques of calculus are essential in the modeling and solving of both mathematical and real-world problems.

Performance Indicators

a. analyzes the concept of limits and applies it to interpret the properties of functions (e.g., continuity, asymptotes)

b. interprets derivatives and definite integrals as limits (e.g., difference quotients, slope, Riemann sums, area)

c. applies the fundamental theorem of calculus

d. applies techniques of differentiation and integration (e.g., product rule, chain rule, *u*-substitution)

e. applies properties of derivatives to analyze the graphs of functions

f. demonstrates knowledge of power series

g. uses derivatives to model and solve mathematical and real-world problems (e.g., rates of change, related rates, optimization)

h. uses integration to model and solve mathematical and real-world problems (e.g., work, applications of antiderivatives)

i. models and solves problems involving first order differential equations (e.g., separation of variables, initial value problems)

COMPETENCY 0005—GEOMETRY AND MEASUREMENT

Performance Expectations

The New York State Mathematics teacher understands the attributes and relationships of geometric objects in diverse contexts and applies the properties of measurement and dimension in modeling situations.

Performance Indicators

a. understands the Pythagorean theorem and its converse (including proofs) and applies the theorem to solve problems in two and three dimensions and in the coordinate plane

b. analyzes and applies properties of rotations, reflections, and translations in the plane and demonstrates understanding of congruence in terms of rigid motions

c. proves and applies theorems about lines and angles, triangles, and parallelograms

d. proves and analyzes geometric constructions

e. demonstrates understanding of similarity in terms of similarity transformations and proves theorems involving similarity

f. applies right triangle trigonometry to solve problems, and applies trigonometry to general triangles (e.g., law of sines, law of cosines)

g. applies theorems about circles and solves measurement problems involving circles (e.g., arc lengths, areas of sectors)

h. translates between geometric descriptions and equations for conic sections (e.g., circles, parabolas, ellipses, hyperbolas)

i. uses coordinates to prove simple geometric theorems algebraically (e.g., slope criteria for parallel and perpendicular lines, properties of polygons)

j. demonstrates understanding of area and volume formulas and Cavalieri's principle, and uses them to model and solve problems

k. identifies relationships between two-dimensional and three-dimensional objects

l. applies geometric concepts in modeling situations (e.g., using shapes to describe objects, applying concepts of density based on area and volume)

m. demonstrates knowledge of non-Euclidean geometry

COMPETENCY 0006—STATISTICS AND PROBABILITY

Performance Expectations

The New York State Mathematics teacher understands that information contained in data is often obscured by variability and uses statistical tools and knowledge of probability to make informed decisions that allow for this variability.

Performance Indicators

a. summarizes and represents data on a single count or measurement variable (e.g., using number lines, dot plots, histograms, and box plots)

b. summarizes and represents data on two categorical and quantitative variables (e.g., using two-way frequency tables and scatter plots)

c. interprets one- and two-variable data presented in a variety of formats (e.g., analyzing data plots in terms of mean, median, interquartile range, standard deviation, and outliers; interpreting shape, center, and spread of data sets; using the mean and standard deviation to fit data to a normal distribution and analyze the fit; analyzing trends; fitting functions to data; plotting and analyzing residuals)

d. interprets correlation coefficients for linear models and distinguishes between correlation and causation

e. demonstrates understanding of random processes, random variables, and probability distributions (e.g., normal, binomial, uniform distributions)

f. evaluates statistical experiments (e.g., making inferences about population parameters from a single random sample)

g. recognizes the purposes of and differences between sample surveys, experiments, and observational studies and makes inferences and justifies conclusions from them

h. demonstrates understanding of independence and conditional probability and uses them to interpret data

i. uses the rules of probability (e.g., addition rule, multiplication rule) and/or permutations and combinations to compute probabilities of compound events in a uniform probability model

j. calculates expected values and uses them to solve problems

k. uses probabilities to evaluate outcomes of decisions

COMPETENCY 0007—PEDAGOGICAL CONTENT KNOWLEDGE

Performance Expectations

The New York State Mathematics teacher effectively applies pedagogical content knowledge across multiple content domains to design instruction to help students achieve a specific learning goal. The teacher analyzes student understanding and identifies potential and apparent student difficulties. The teacher applies knowledge of how students learn to develop an effective instructional strategy that includes multiple ways of representing mathematical concepts and procedures that will facilitate development of students' skills and their achievement of the desired learning goal.

Performance Indicators

a. identifies the skills and conceptual understanding necessary for students to achieve a specific new learning goal

b. demonstrates knowledge of methods for assessing student readiness for a specific new learning goal

c. demonstrates knowledge of ways to connect students' prior learning to the new learning goal

d. promotes coherence by connecting learning across the mathematical domains

e. describes an appropriate and effective instructional strategy that includes multiple representations of essential/difficult concepts

f. demonstrates knowledge of methods for assessing students' progress during the lesson toward achieving the learning goal